Data-driven imaging with second-order traveltime approximations

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EAGE/SEG Summer TRIESTE 2003 Research Workshop

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- Motivation & data examples

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- Basic concepts

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- Possible derivations

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- Hypothetical experiments

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- Hypothetical experiments
- Applications of wavefield attributes

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- Conclusions

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- Hypothetical experiments
- Applications of wavefield attributes
- Conclusions
- Outlook

Motivation

Model-based approaches:

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- sensitive to model errors

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Data-driven approaches:

- interval velocity model determination is postponed
- robust methods
- however, classic data-driven approaches
  - use only a subset of available data, thus no optimum S/N ratio
  - provide little information for later inversion
  - data-driven aspects usually not fully exploited

Motivation

Common-Reflection-Surface (CRS) stack:

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- minimum a priori information required

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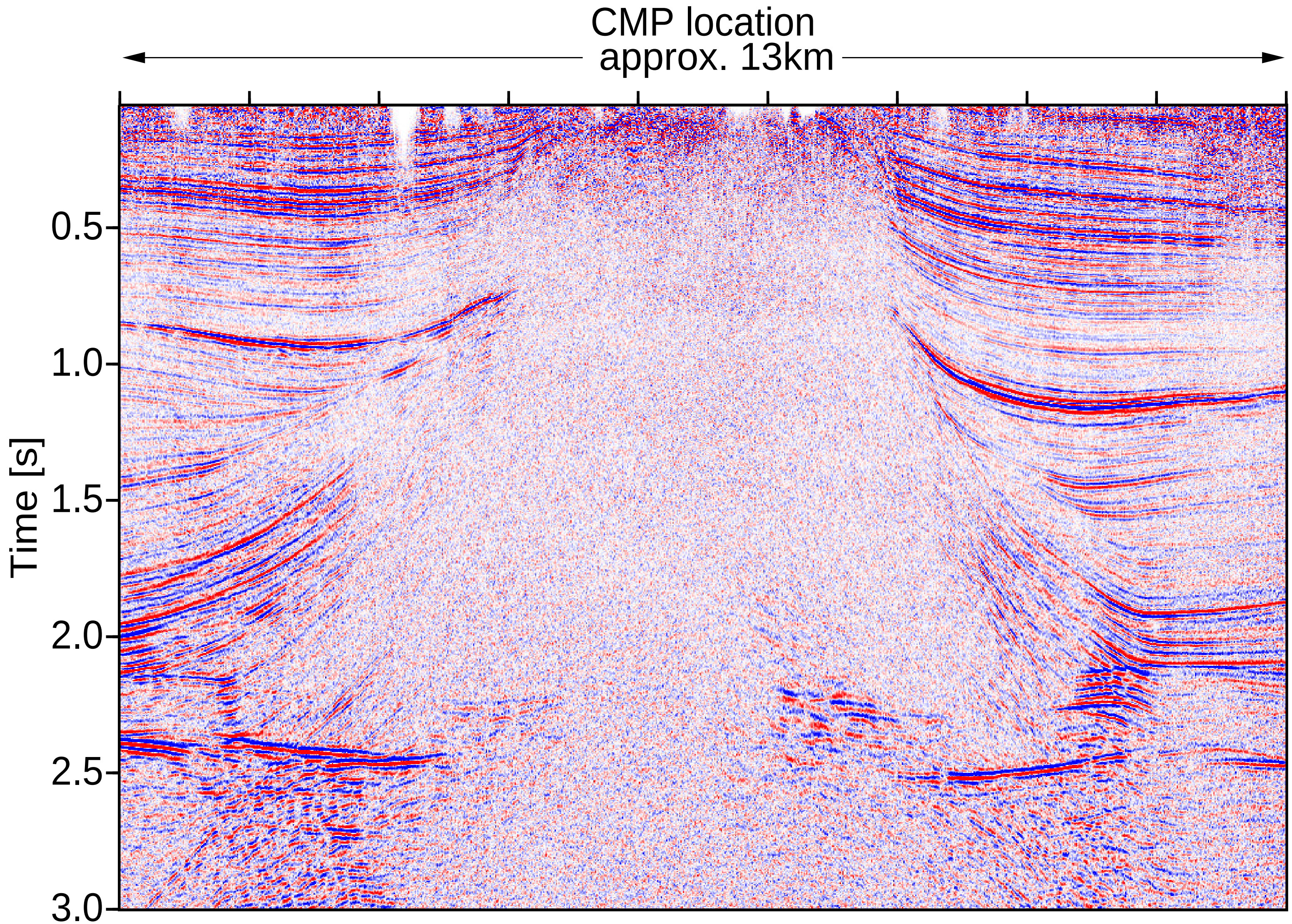
Common-Reflection-Surface (CRS) stack:

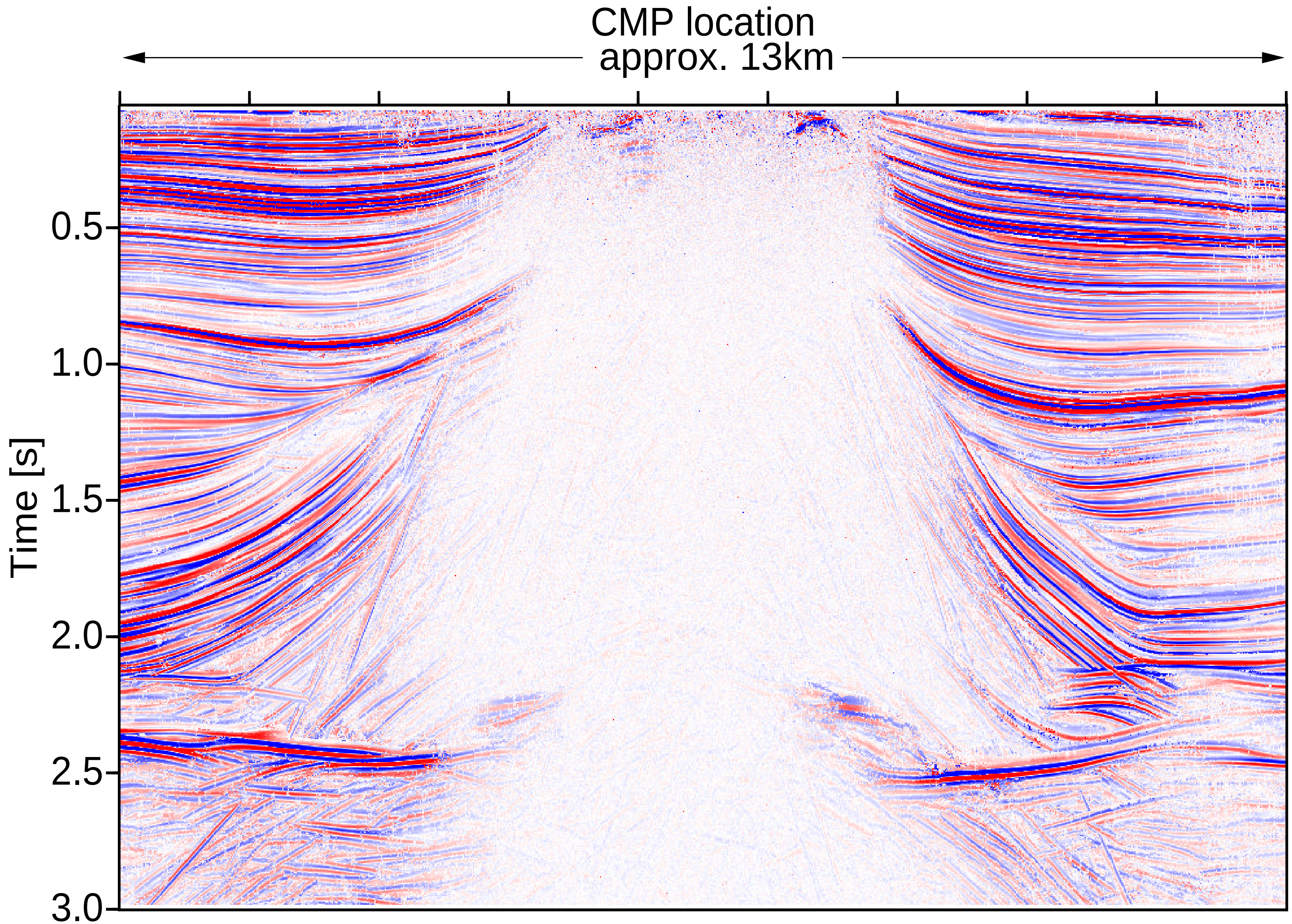
- extension of concepts of classic data-driven approaches
- full use of available data
- minimum a priori information required
- fully data-driven application

Data example A



2-D NMO/DMO/stack – from Müller (1999)

Data example A



2-D CRS stack – from Müller (1999)

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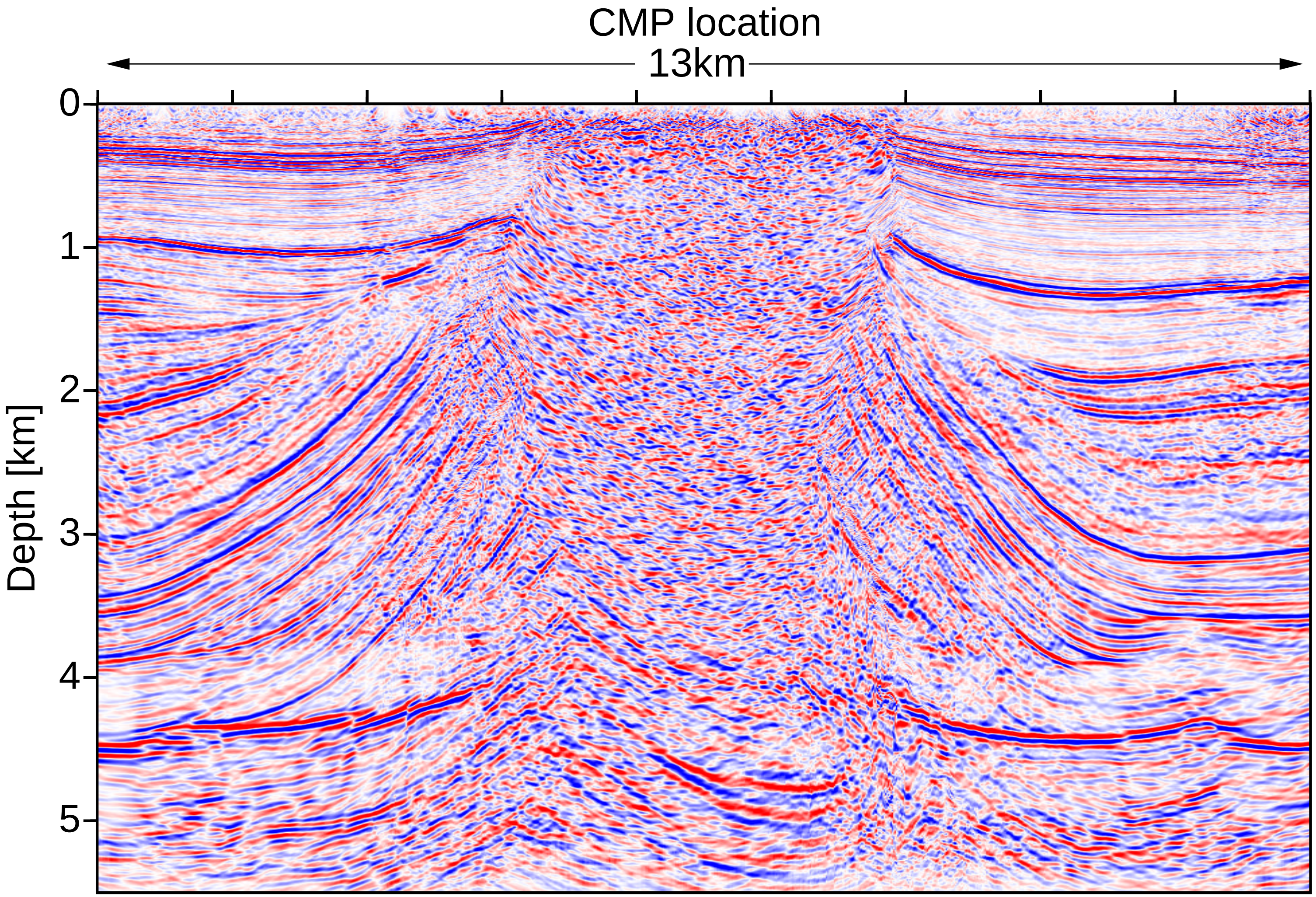
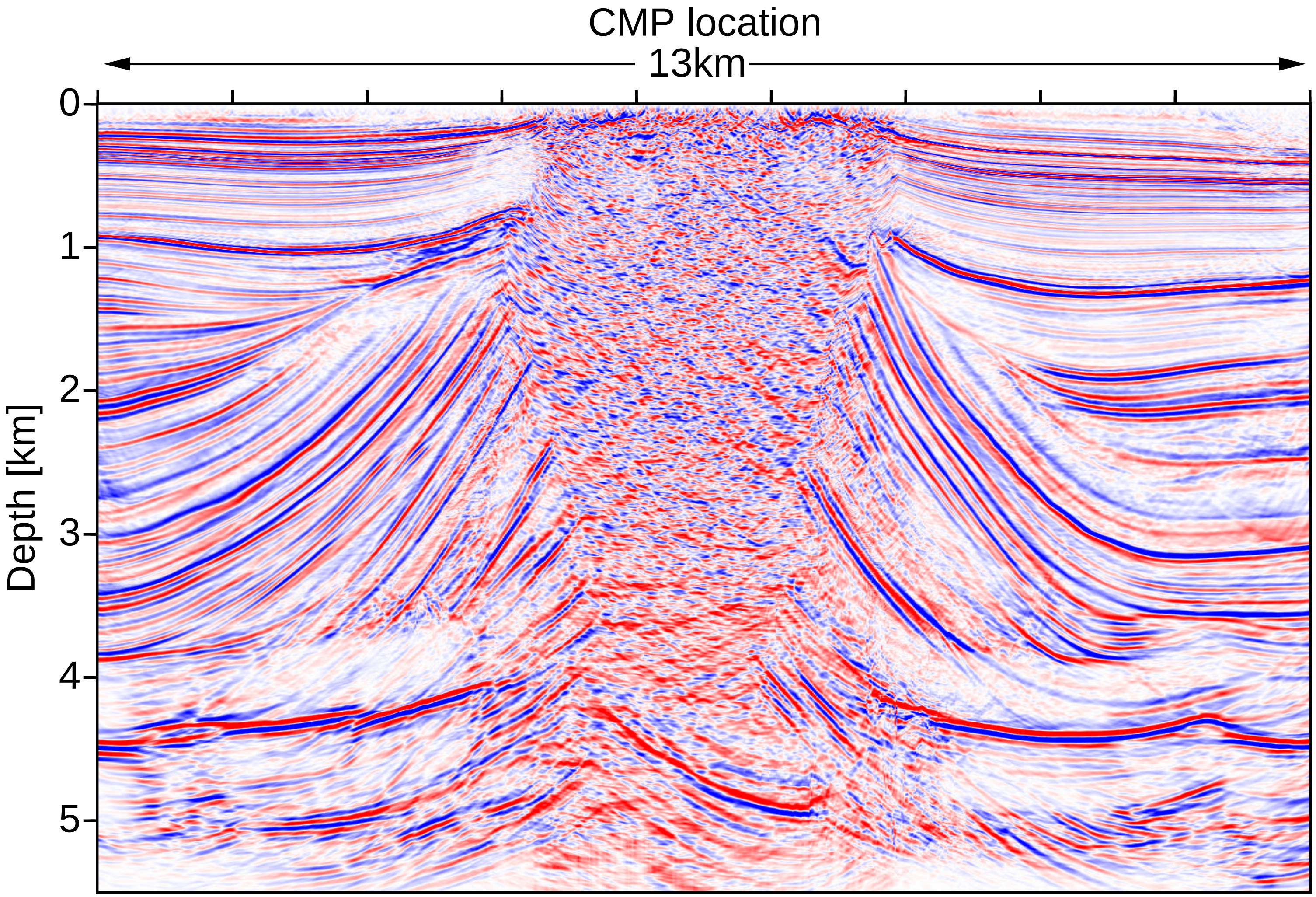


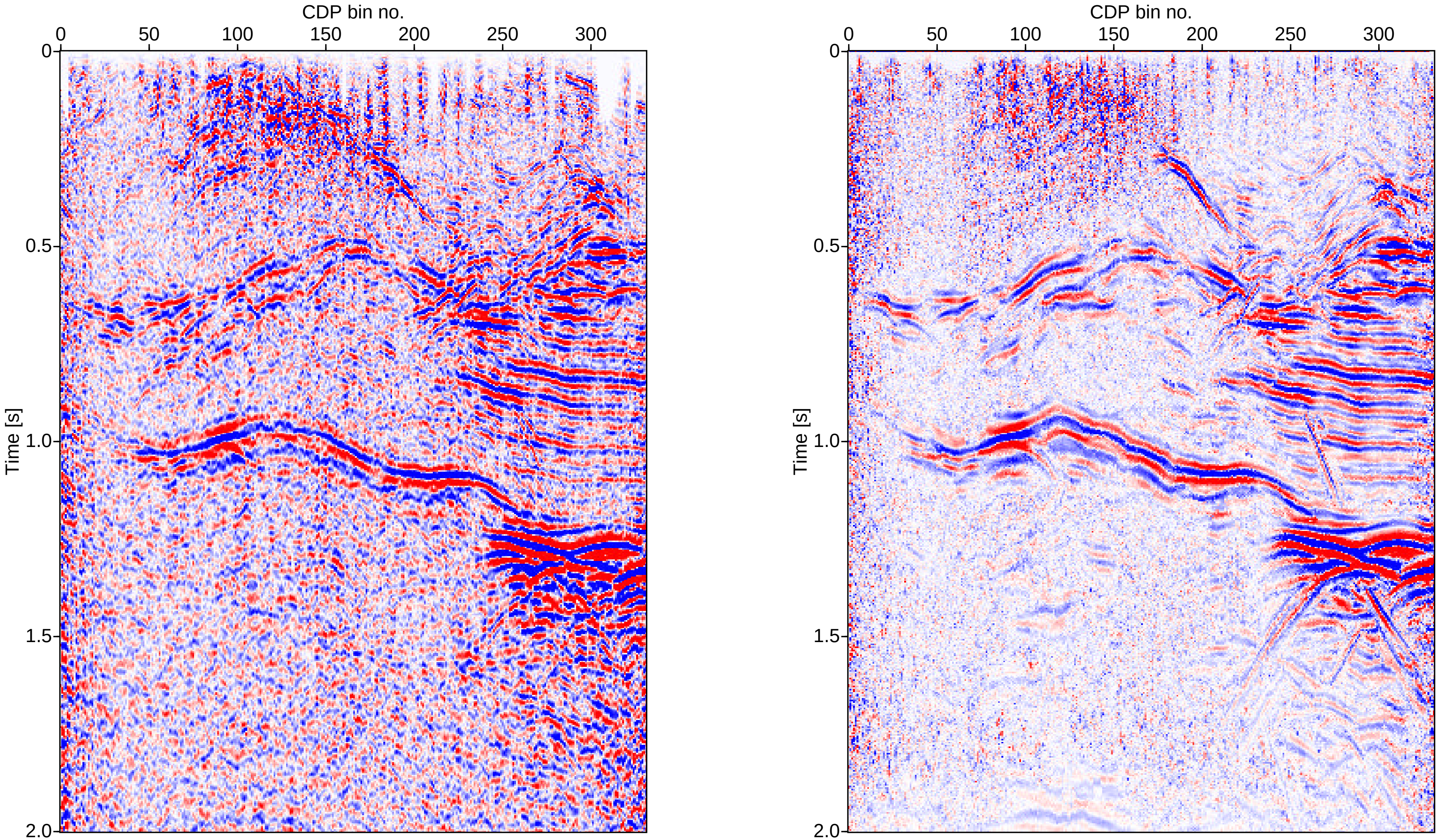
NMO/DMO/stack/poststack migration – from Müller (1999)

Data example A



2-D CRS/poststack migration – from Müller (1999)

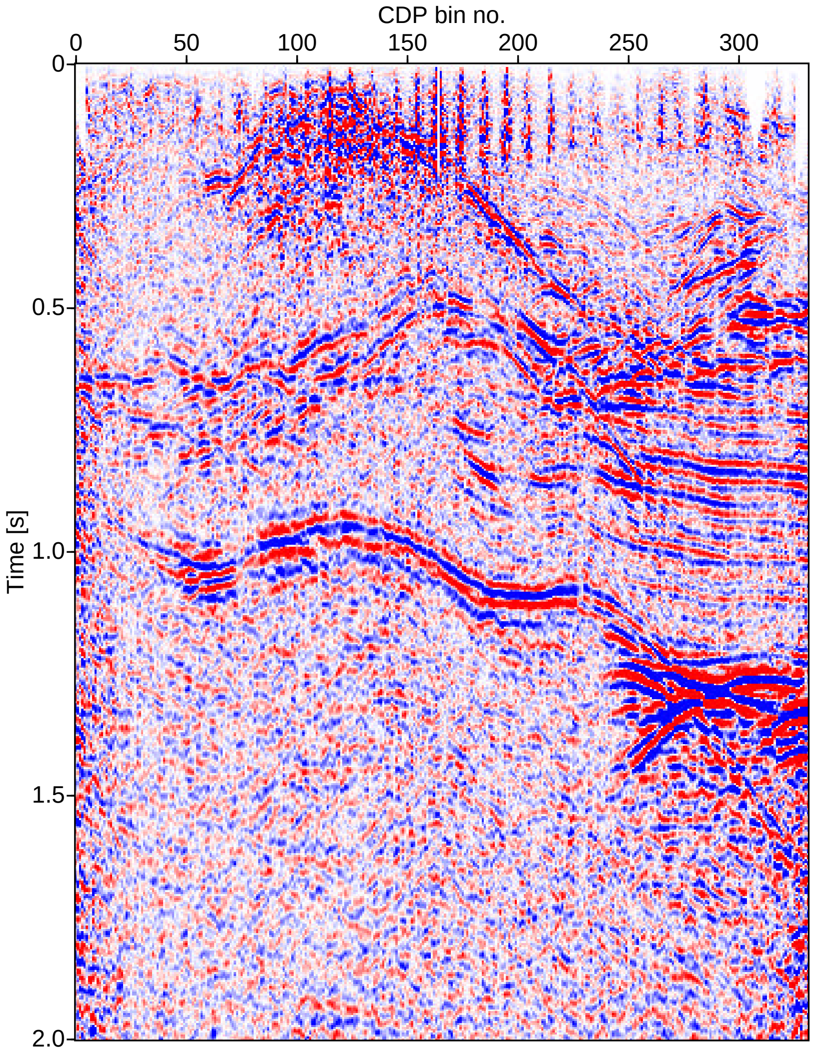
Data example B

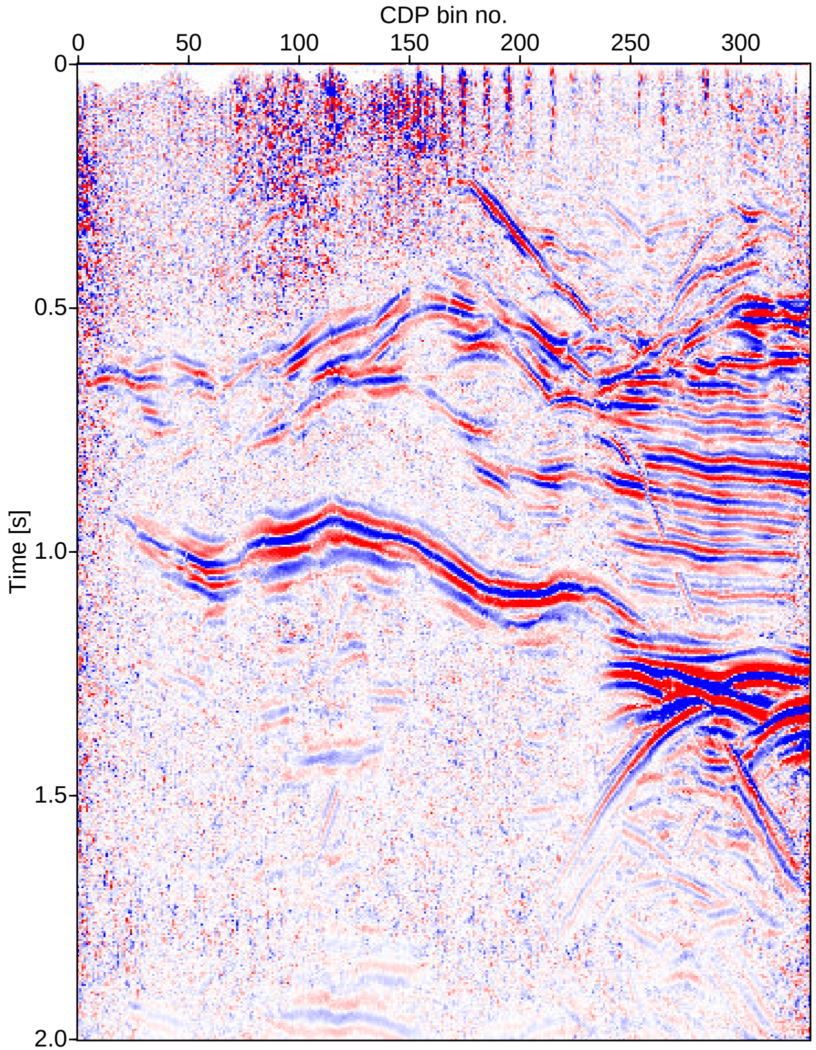


NMO/DMO/stack vs. CRS stack – 3-D data, inline A

From Bergler et. al (2002). Data courtesy of ENI E & P Division.

Data example B

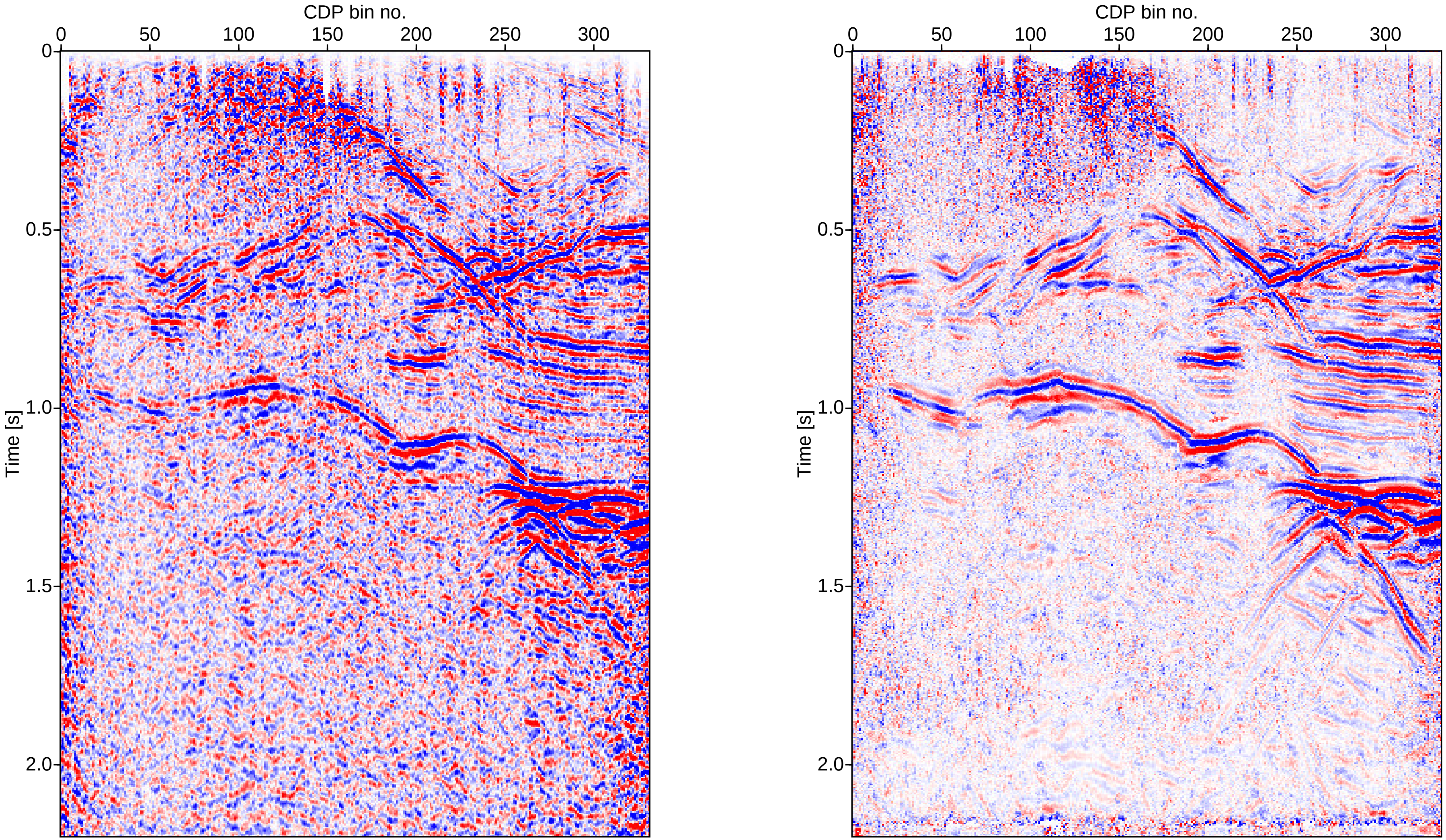




NMO/DMO/stack vs. CRS stack – 3-D data, inline B

From Bergler et. al (2002). Data courtesy of ENI E & P Division.

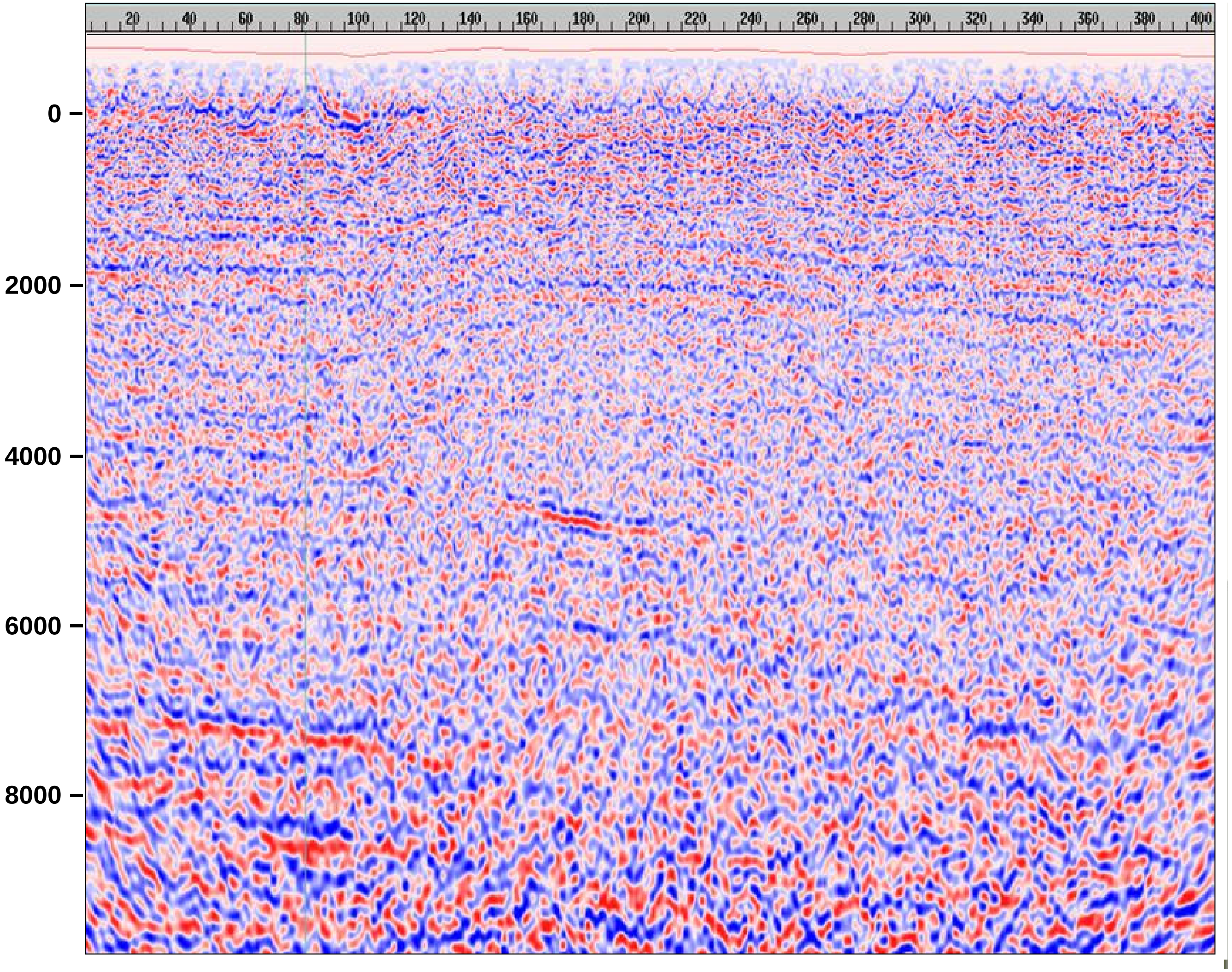
Data example B



NMO/DMO/stack vs. CRS stack – 3-D data, inline C

From Bergler et. al (2002). Data courtesy of ENI E & P Division.

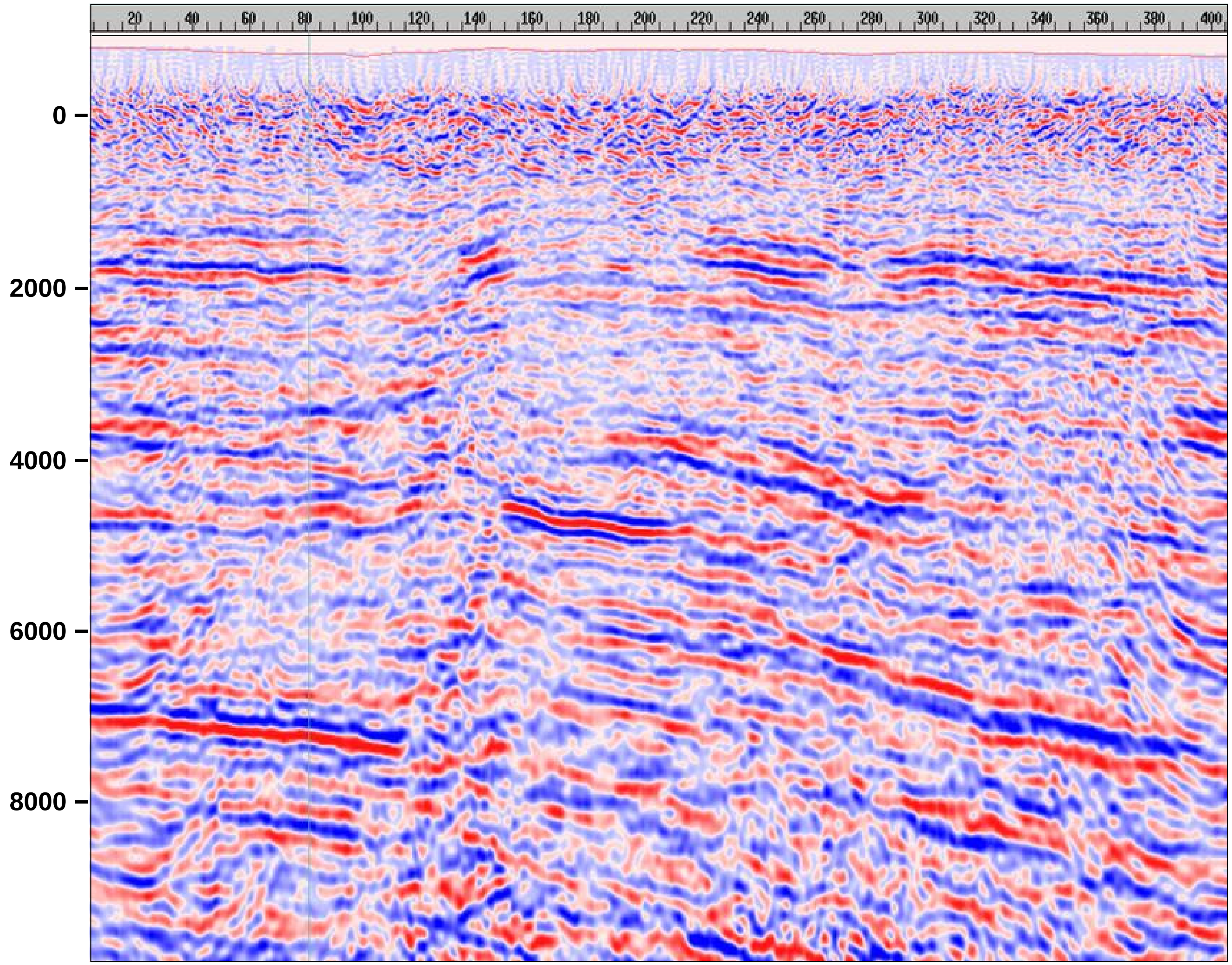
Data example C



Conventional 3-D prestack depth migration

Courtesy of ENI E & P Division

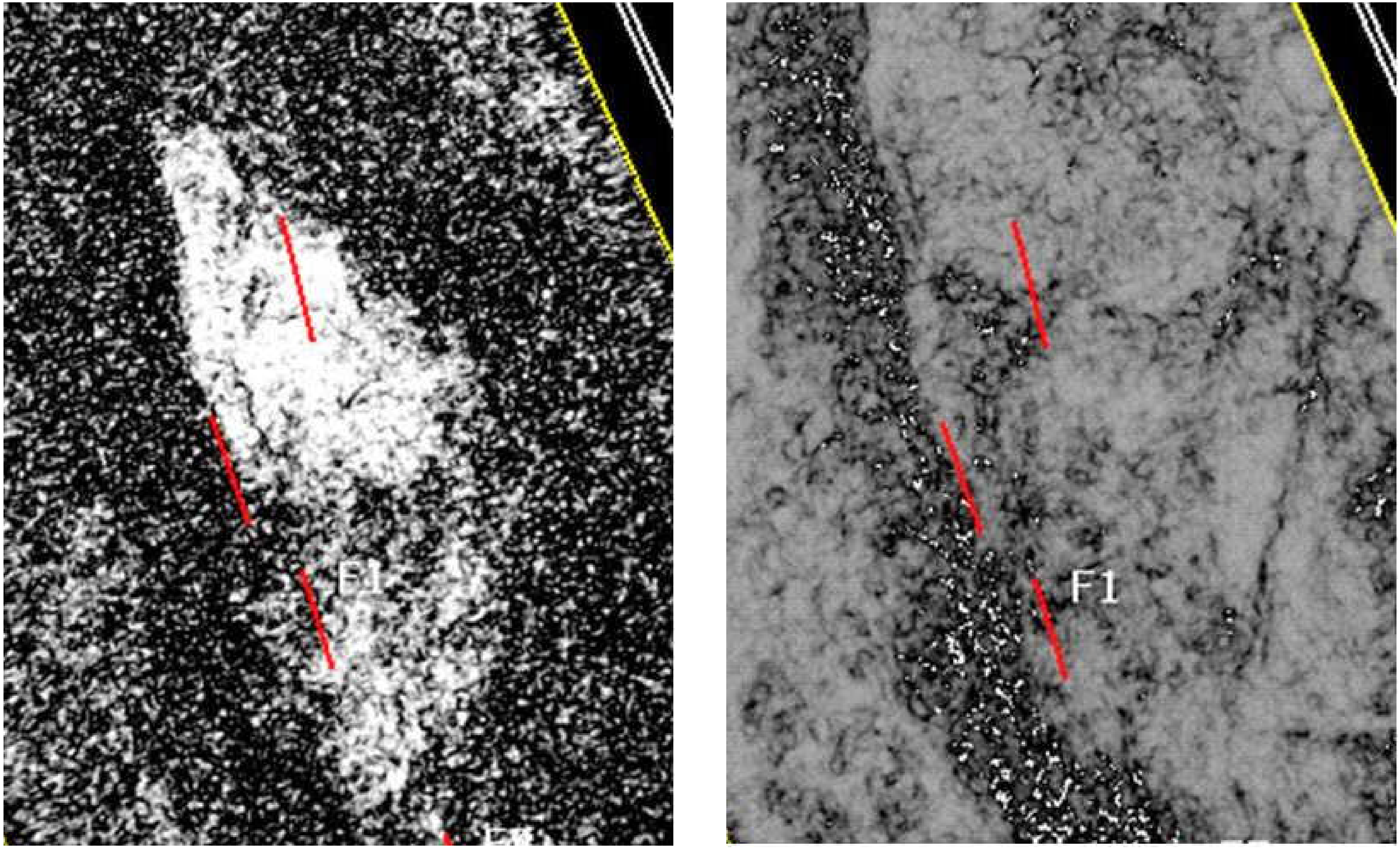
Data example C



3-D poststack depth migration of CRS stack

Courtesy of ENI E & P Division

Data example C



depth slices of coherence images: conventional vs. CRS-based

Courtesy of ENI E & P Division

Data examples

More data examples:

Presentation by Cardone et al.

Presentation by Trappe et al.

in this session

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  - or in the depth domain at the acquisition surface
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  - or in the depth domain at the acquisition surface
    ➥ properties of hypothetical wavefronts,

  both linked by the near-surface velocity v_0.

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  ↪ generalized multi-dimensional velocity analysis

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- a simulated section for an arbitrarily chosen configuration
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- an associated coherence section
  ➥ identification of events, reliability of attributes

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- paraxial ray theory, i. e., assumption of a linear relation between the properties of neighboring rays
- geometrical optics using the concept of object and image points (2-D case only)
- pragmatic way: second-order expansion of traveltime, initially without physical interpretation

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Prestack data:

(hyper-)volume $p(t, \vec{m}, \vec{h})$ with up to five dimensions

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t	time
$\vec{m} = \begin{pmatrix} m_x \\ m_y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} g_x + s_x \\ g_y + s_y \end{pmatrix}$	midpoint vector
$\vec{h} = \begin{pmatrix} h_x \\ h_y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} g_x - s_x \\ g_y - s_y \end{pmatrix}$	half-offset vector

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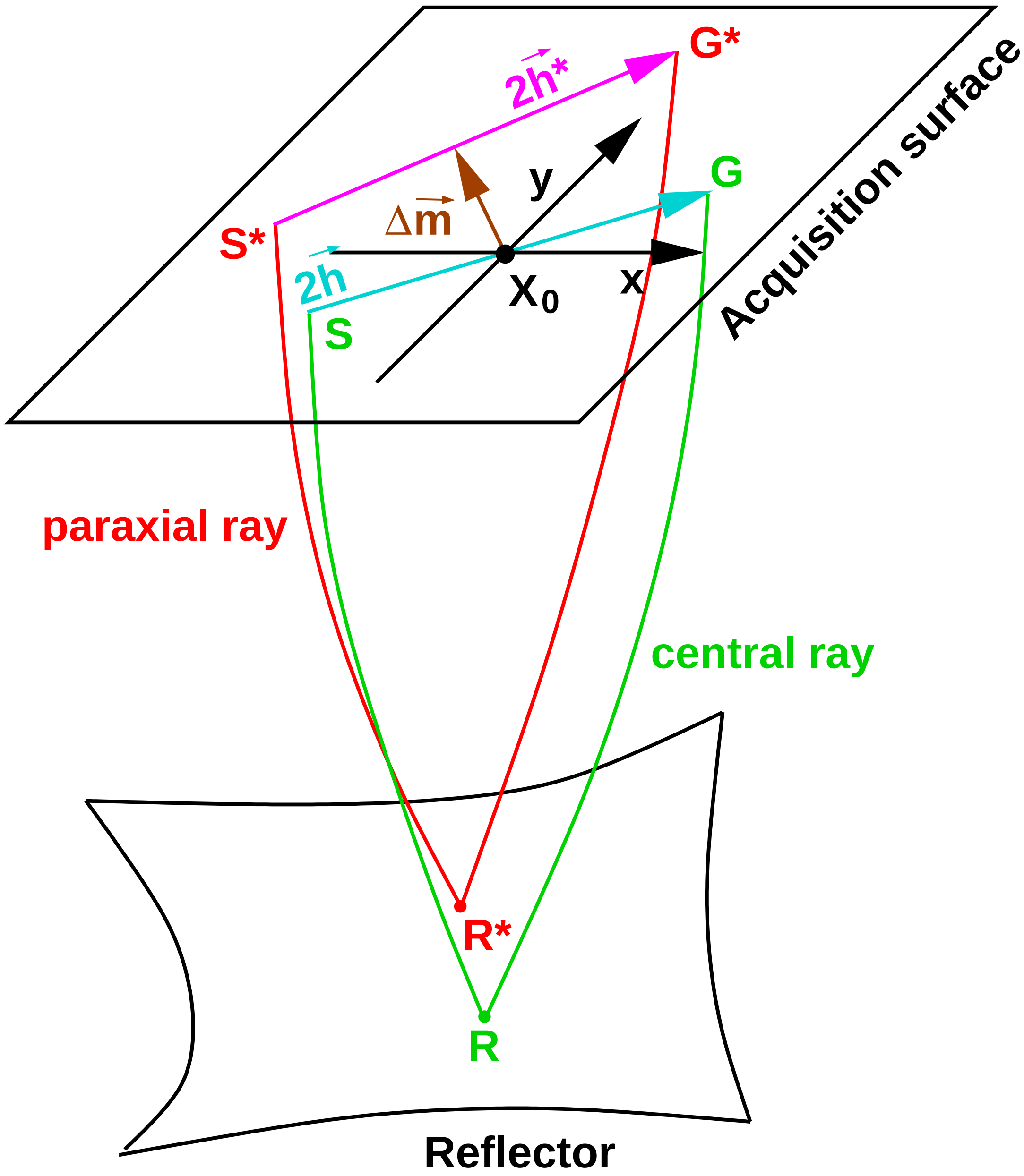
Reflection event:

(hyper-)surface $t\left(\vec{m}, \vec{h}\right)$ in the prestack data

Central and paraxial rays

Assumed to be known:
traveltime $t\left(\vec{m},\vec{h}\right)$ along central ray (SRG)



$$\Delta\vec{h} = \vec{h}^* - \vec{h}$$

Central and paraxial rays

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How to approximate
$t\left(\vec{m}+\Delta\vec{m},\vec{h}+\Delta\vec{h}\right)$ along
paraxial ray (S*R*G*)?

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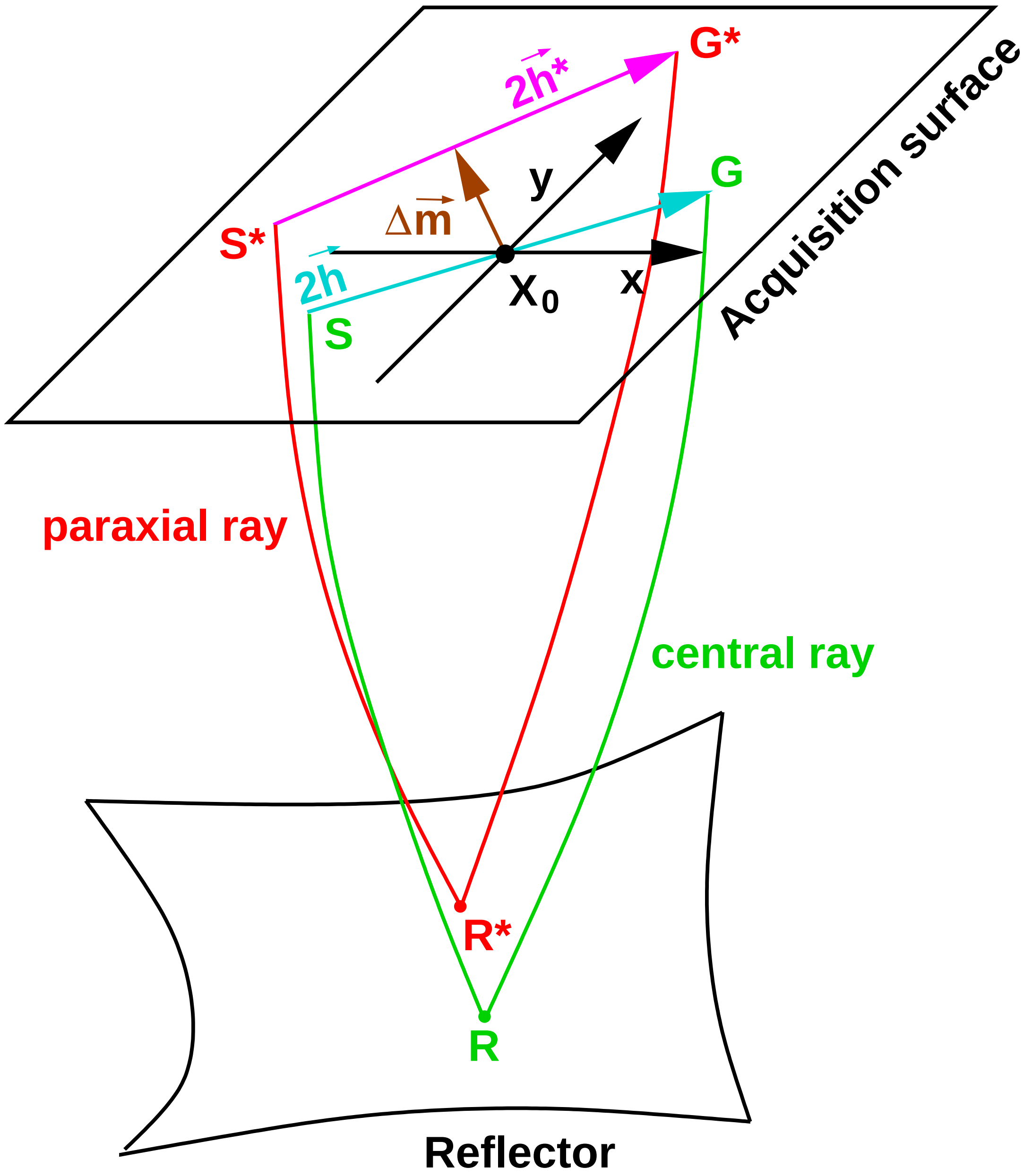

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➥ Taylor expansion

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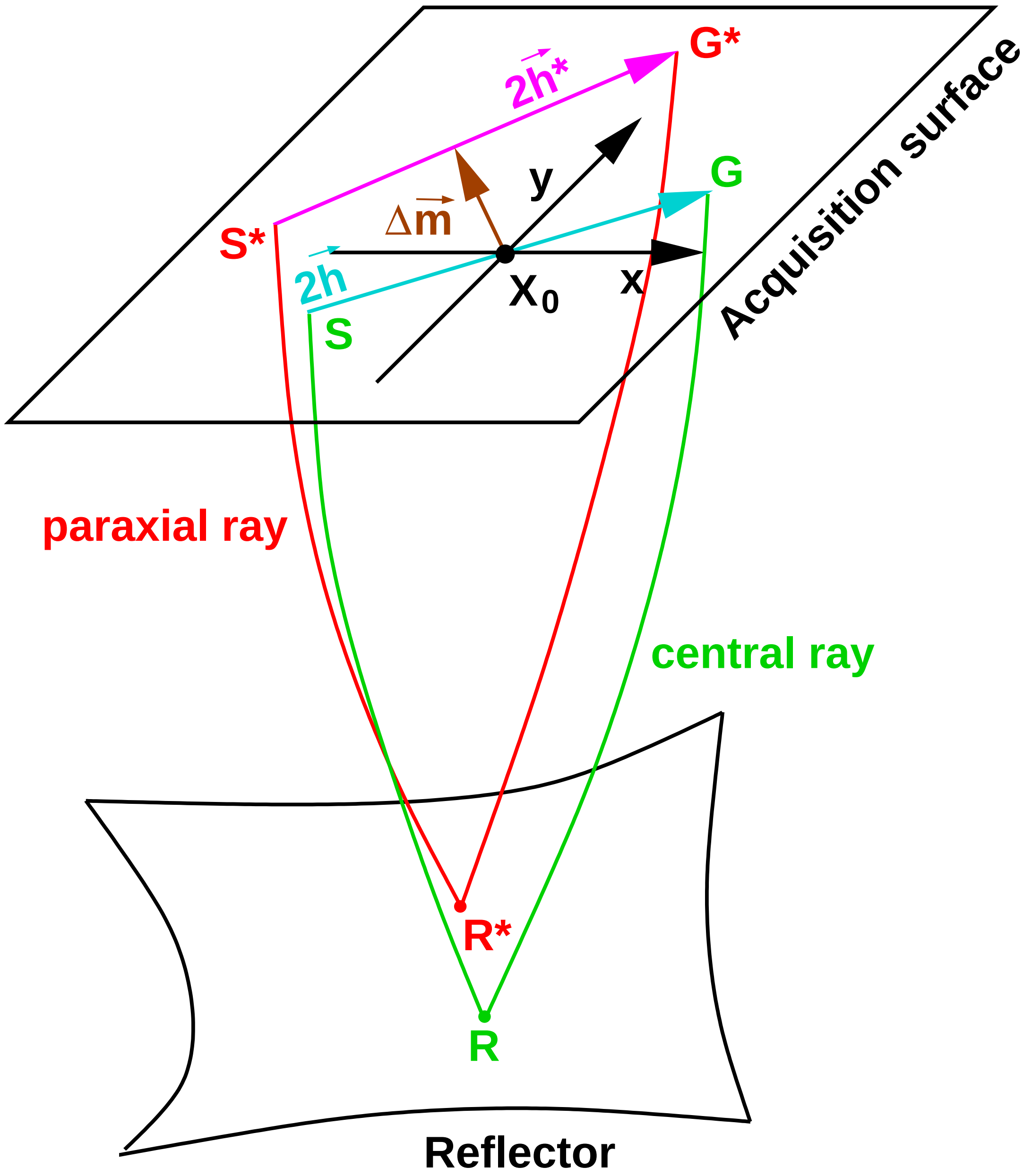

Pragmatic approach

Taylor expansion up to second order – general case

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$$t\left(\vec{m}+\Delta\vec{m},\vec{h}+\Delta\vec{h}\right)\approx$$

$$t\left(\vec{m},\vec{h}\right)+\frac{\partial t}{\partial m_x}\Delta m_x+\frac{\partial t}{\partial m_y}\Delta m_y+\frac{\partial t}{\partial h_x}\Delta h_x+\frac{\partial t}{\partial h_y}\Delta h_y$$

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Pragmatic approach

Special case: Marine acquisition, single azimuth

$$t\left(\vec{m}+\Delta\vec{m},\vec{h}+\Delta\vec{h}\right)\approx$$

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Pragmatic approach

Special case: 2-D acquisition

$$t\left(\vec{m}+\Delta\vec{m},\vec{h}+\Delta\vec{h}\right)\approx$$

$$t\left(\vec{m},\vec{h}\right)+\frac{\partial t}{\partial m_x}\Delta m_x+\frac{\partial t}{\partial m_y}\Delta m_y+\frac{\partial t}{\partial h_x}\Delta h_x+\frac{\partial t}{\partial h_y}\Delta h_y$$

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Pragmatic approach

General case

$$t\left(\vec{m}+\Delta\vec{m},\vec{h}+\Delta\vec{h}\right) \approx$$

$$t\left(\vec{m},\vec{h}\right) + \frac{\partial t}{\partial m_x}\Delta m_x + \frac{\partial t}{\partial m_y}\Delta m_y + \frac{\partial t}{\partial h_x}\Delta h_x + \frac{\partial t}{\partial h_y}\Delta h_y$$

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Pragmatic approach

Special case: zero-offset simulation

$$t\left(\vec{m}+\Delta\vec{m},\vec{h}+\Delta\vec{h}\right)\approx$$

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Pragmatic approach

Special case: zero-offset simulation, marine case

$$t\left(\vec{m}+\Delta\vec{m},\vec{h}+\Delta\vec{h}\right)\approx$$

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Pragmatic approach

Special case: zero-offset simulation, 2-D acquisition

$$
t\left(\vec{m}+\Delta\vec{m},\vec{h}+\Delta\vec{h}\right) \approx
$$

$$
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$$

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Special case: ZO simulation, 2-D, CMP gathers only

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$$t\left(\vec{m},\vec{h}\right)+\frac{\partial t}{\partial m_x}\Delta m_x+\frac{\partial t}{\partial m_y}\Delta m_y+\frac{\partial t}{\partial h_x}\Delta h_x+\frac{\partial t}{\partial h_y}\Delta h_y$$

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$$+\frac{\partial^2 t}{\partial m_y \partial h_x}\Delta m_y \Delta h_x+\frac{\partial^2 t}{\partial m_y \partial h_y}\Delta m_y \Delta h_y+\frac{\partial^2 t}{\partial h_x \partial h_y}\Delta h_x \Delta h_y$$

Pragmatic approach

Preliminary conclusions:

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- Hyperbolic approximations can be obtained by squaring and neglecting higher order terms.
- We need a physical interpretation of the derivatives
  - to identify hidden dependencies,
  - to understand which values are physically reasonable,
  - and to make use of the derivatives for various purposes.

Physical interpretation

Simplest case: 2-D acquisition, zero-offset

$$t\left(x_m, h\right) = t_0 + \frac{\partial t}{\partial x_m}\left(x_m - x_0\right) + \frac{1}{2}\left[\frac{\partial^2 t}{\partial x_m^2}\left(x_m - x_0\right)^2 + \frac{\partial^2 t}{\partial h^2}h^2\right]$$

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Horizontal slowness:

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$$p_x = \frac{1}{2}\frac{\partial t}{\partial x_m}\Bigg|_{\left(x_m = x_0,\, h = 0\right)} = |\vec{p}| \sin\alpha$$

$\vec{p}$ slowness vector

α emergence angle

v_0 near-surface velocity

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Curvature of “zero-offset wavefront”:

$$K_N = \left.\frac{\partial^2 t}{\partial x_m^2}\right|_{\left(x_m=x_0,\,h=0\right)}$$

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Curvature of “zero-offset wavefront”:

$$K_N = \frac{v_0}{2} \frac{1}{\cos^2\alpha} \frac{\partial^2 t}{\partial x_m^2}\Bigg|_{\left(x_m = x_0, h = 0\right)}$$

A “zero-offset wavefront”, also called normal wavefront, can be obtained from an exploding reflector experiment.

Physical interpretation

Simplest case: 2-D acquisition, zero-offset

$$t(x_m, h) = t_0 + \frac{\partial t}{\partial x_m}(x_m - x_0) + \frac{1}{2}\left[\frac{\partial^2 t}{\partial x_m^2}(x_m - x_0)^2 + \frac{\partial^2 t}{\partial h^2} h^2\right]$$

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Problem: no simple physical experiment available!

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However: up to second order, CMP traveltimes and zero-offset diffraction traveltimes coincide (NIP wave theorem, Hubral 1983).

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However: up to second order, CMP traveltimes and zero-offset diffraction traveltimes coincide (NIP wave theorem, Hubral 1983).

➥ In analogy to the exploding reflector experiment, a exploding reflection point experiment approximates the “CMP wavefront”.

Physical interpretation

Simplest case: 2-D acquisition, zero-offset

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$$K_{NIP}=\frac{1}{2}\frac{v_0}{\cos^2\alpha}\frac{\partial^2 t}{\partial h^2}\bigg|_{\left(x_m=x_0,\,h=0\right)}$$

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An exploding reflection-point experiment yields the so-called normal-incidence-point (NIP) wavefront.

Physical interpretation

Replacing all derivatives, we obtain

$$t\left(x_m, h\right) = t_0 + \frac{2\sin\alpha}{v_0}\left(x_m - x_0\right) + \frac{\cos^2\alpha}{v_0}\left[K_N\left(x_m - x_0\right) + K_{NIP}\, h^2\right]$$

in terms of *kinematic wavefield attributes*.

Physical interpretation

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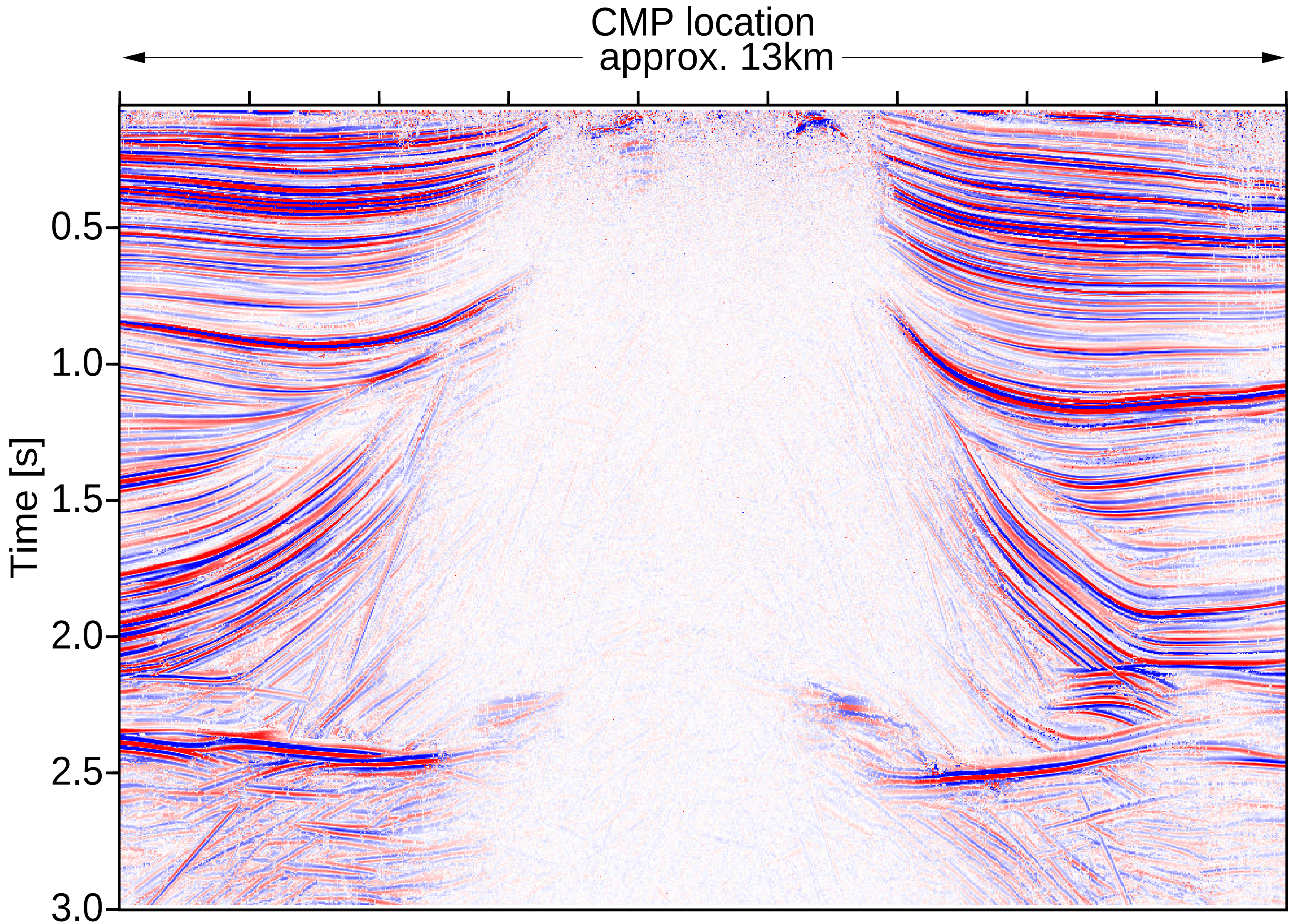
$$t\left(x_m,h\right)=t_0+\frac{2\sin\alpha}{v_0}\left(x_m-x_0\right)+\frac{\cos^2\alpha}{v_0}\left[K_N\left(x_m-x_0\right)+K_{NIP}\,h^2\right]$$

in terms of *kinematic wavefield attributes*.

Accordingly, the hyperbolic counterpart reads

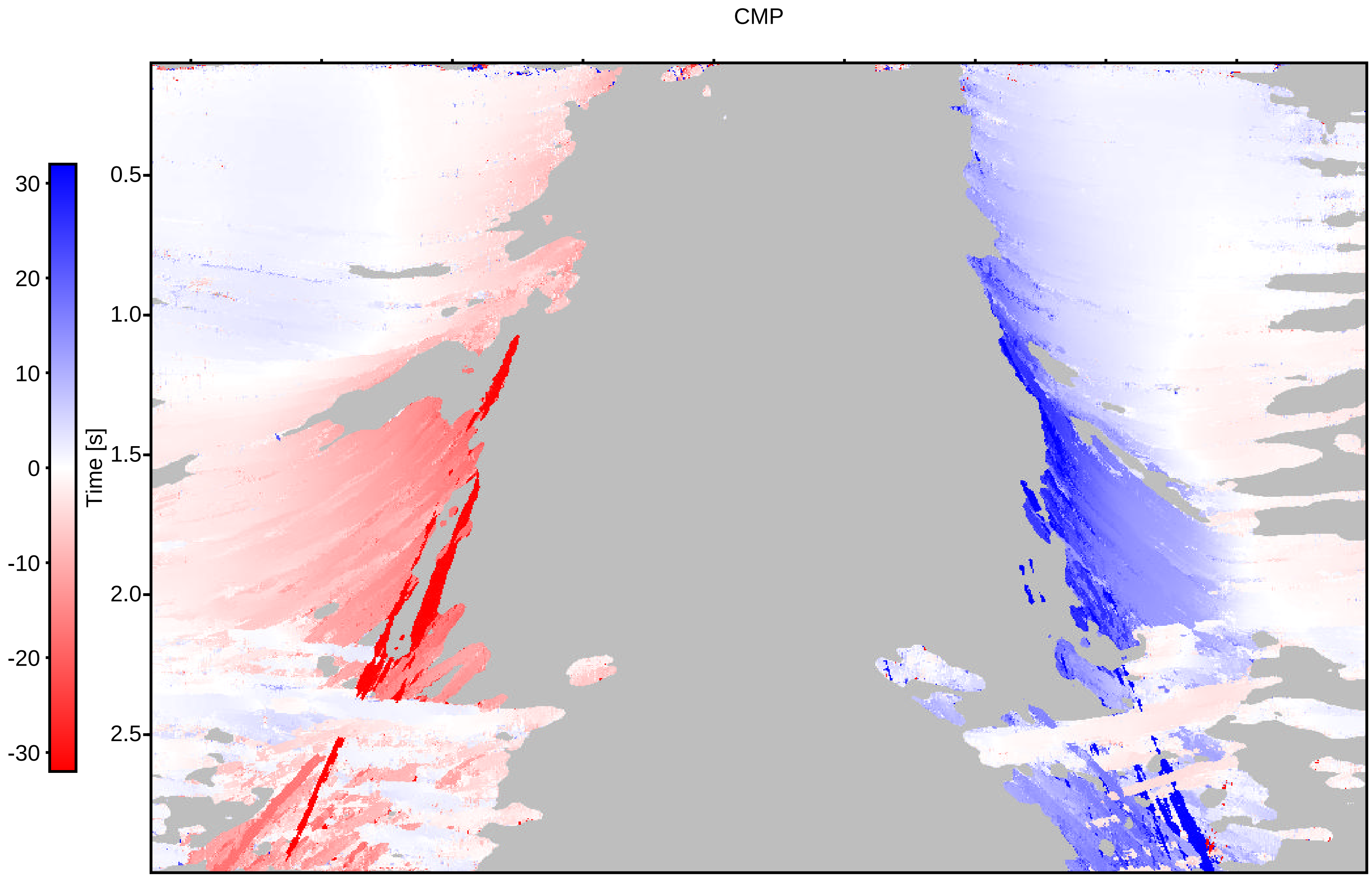
$$t^2\left(x_m,h\right)\approx\tilde{t}^{\,2}\left(x_m,h\right)=\left[t_0+\frac{2\sin\alpha}{v_0}\left(x_m-x_0\right)\right]^2+\frac{2\,t_0\cos^2\alpha}{v_0}\left[K_N\left(x_m-x_0\right)^2+K_{NIP}\,h^2\right].$$

Data example A



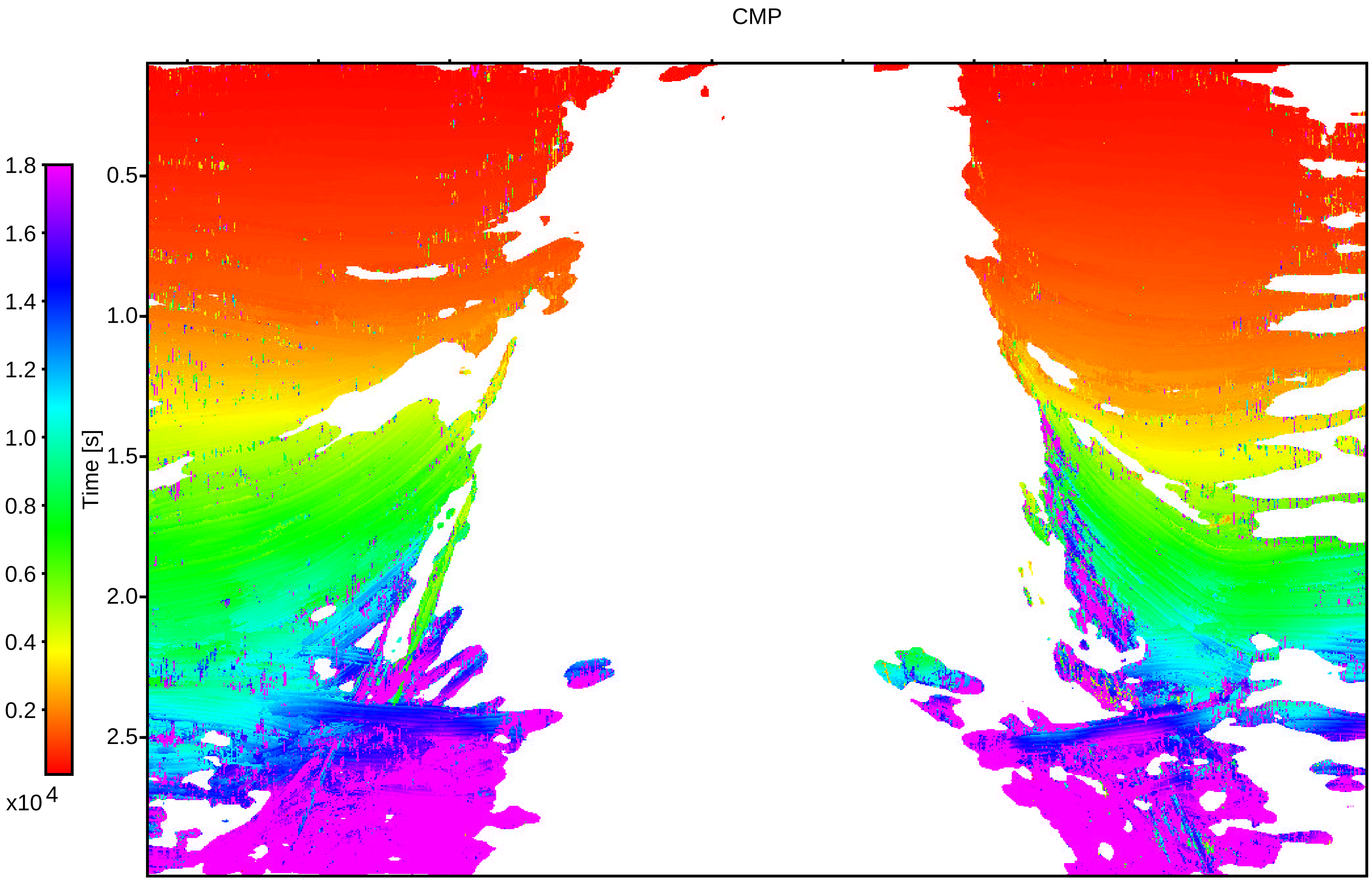
2-D CRS stack – from Müller (1999)

Data example A



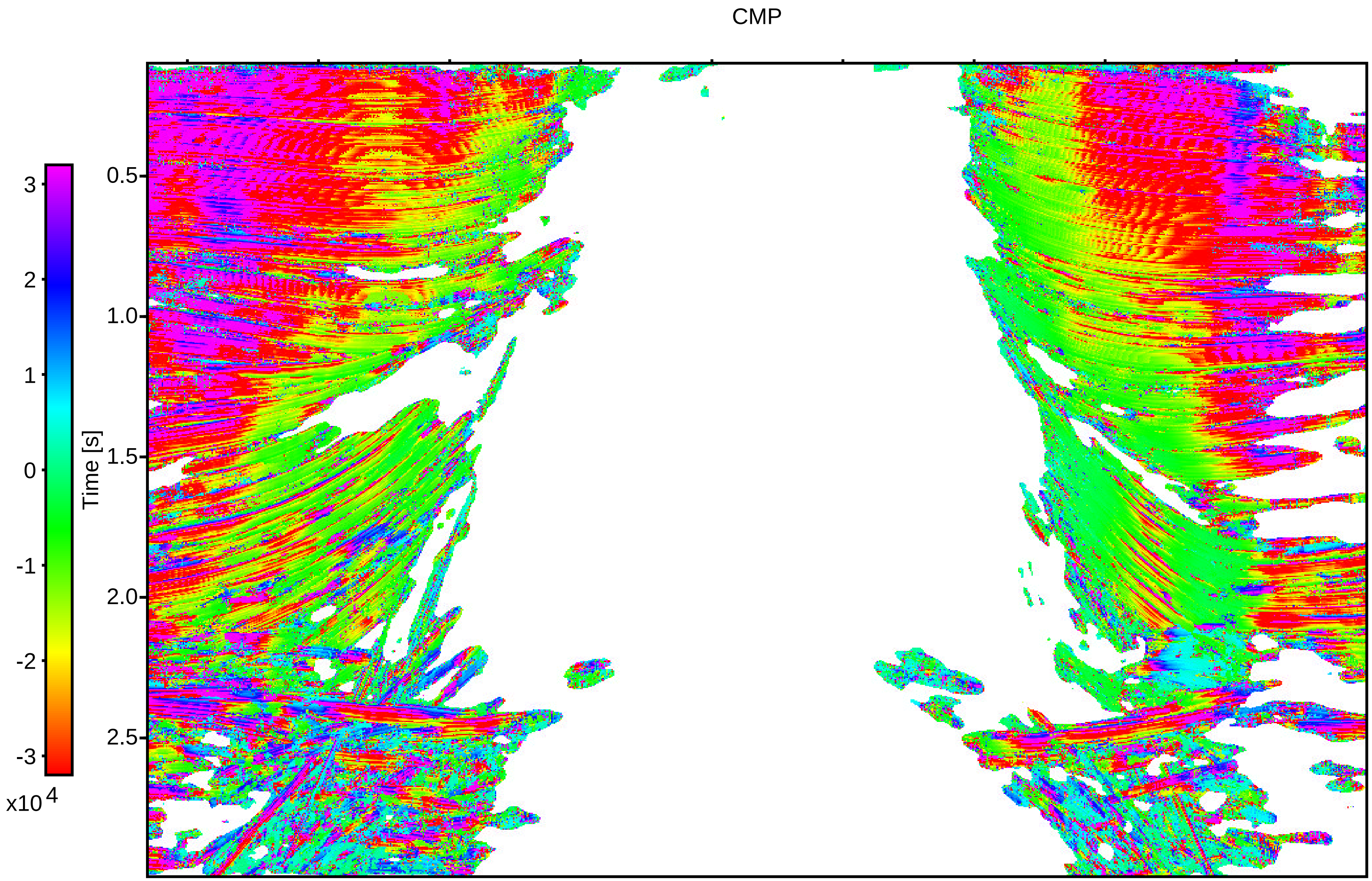
Emergence angle α $[°]$

Data example A



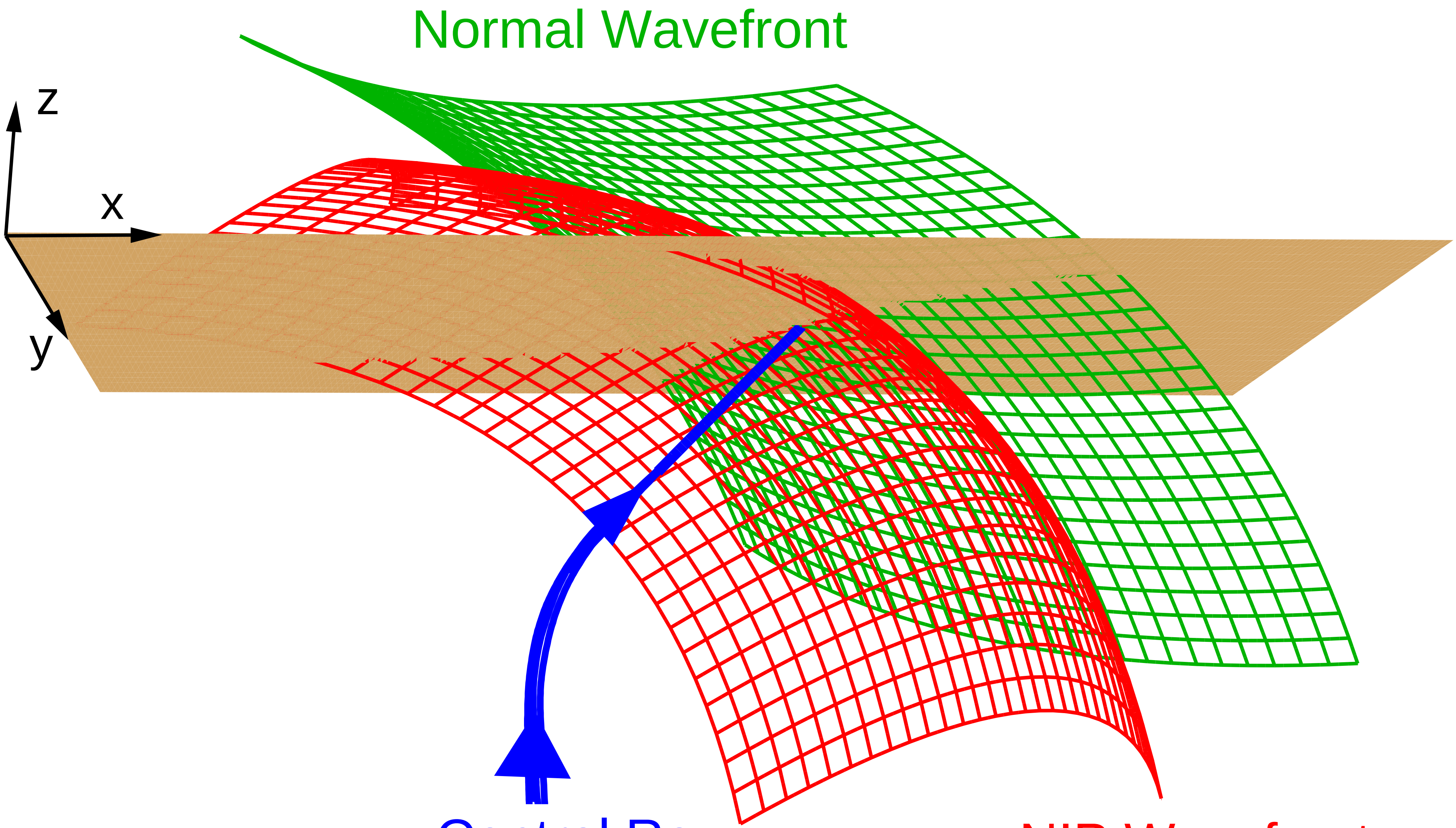
Radius of curvature of NIP wavefront [m]

Data example A



Radius of curvature of normal wavefront [m]

From 2-D to 3-D



From 2-D to 3-D

From scalar curvatures to curvature matrices:

From 2-D to 3-D

From scalar curvatures to curvature matrices:

$$K_{NIP} \mapsto \mathbf{K}_{NIP} = \frac{v_0}{2}\,\mathbf{T}^{\mathrm{T}} \begin{pmatrix} \frac{\partial^2 t}{\partial h_x^2} & \frac{\partial^2 t}{\partial h_x \partial h_y} \\ \frac{\partial^2 t}{\partial h_y \partial h_x} & \frac{\partial^2 t}{\partial h_y^2} \end{pmatrix} \mathbf{T}$$

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From 2-D to 3-D

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From scalar horizontal slowness to horizontal slowness vector:

$$p_x \mapsto \begin{pmatrix} p_x \\ p_y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \frac{\partial t}{\partial m_x} \\ \frac{\partial t}{\partial m_y} \end{pmatrix}$$

Finite-offset vs. zero-offset case

Zero-offset case:

Finite-offset vs. zero-offset case

Zero-offset case:

- NIP and normal wavefronts from one-way experiments (exploding reflector and exploding reflection point)

Finite-offset vs. zero-offset case

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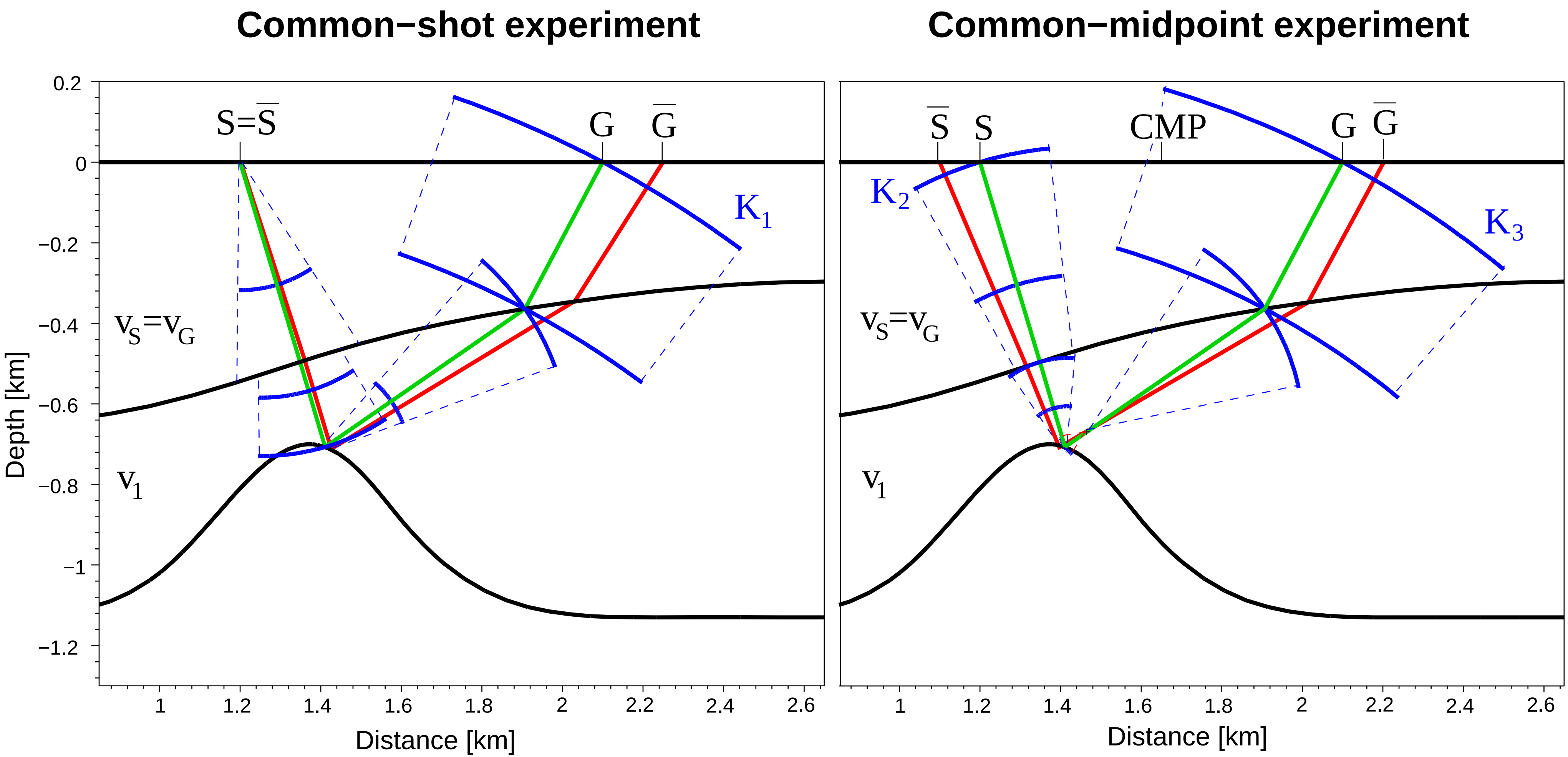
Finite-offset vs. zero-offset case

Hypothetical experiments in the finite-offset case



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☞ presentation by Bergler and Hubral in this session ☜

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    - downward propagation of NIP wavefronts until $R_{NIP} = 0 \wedge t_0 = 0$

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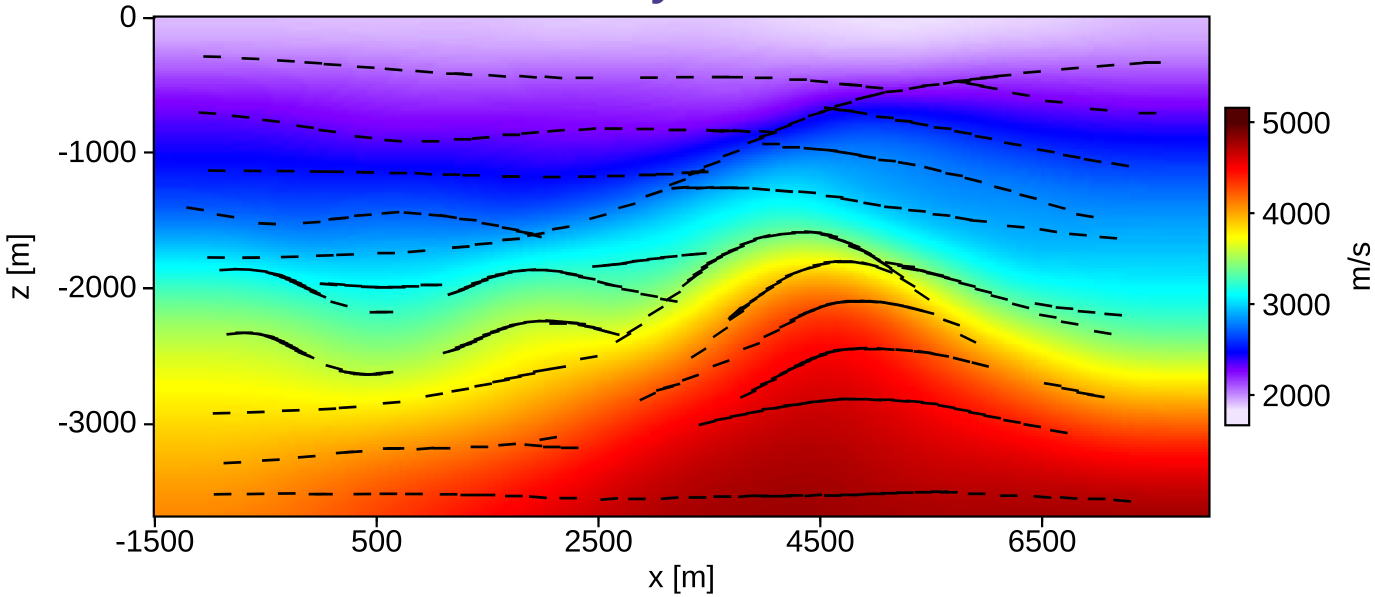
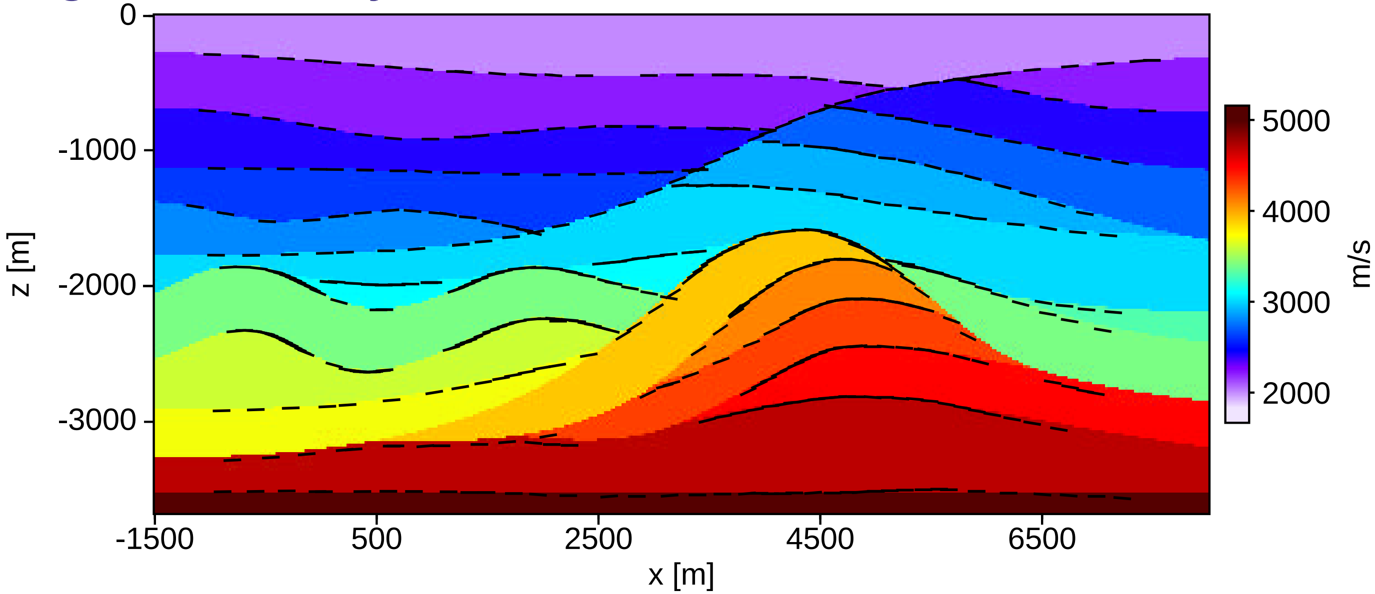
    - iterative model updates to minimize misfit

Reconstructed vs. original model

Reconstructed velocity and reflector elements



Original velocity and reconstructed reflector elements



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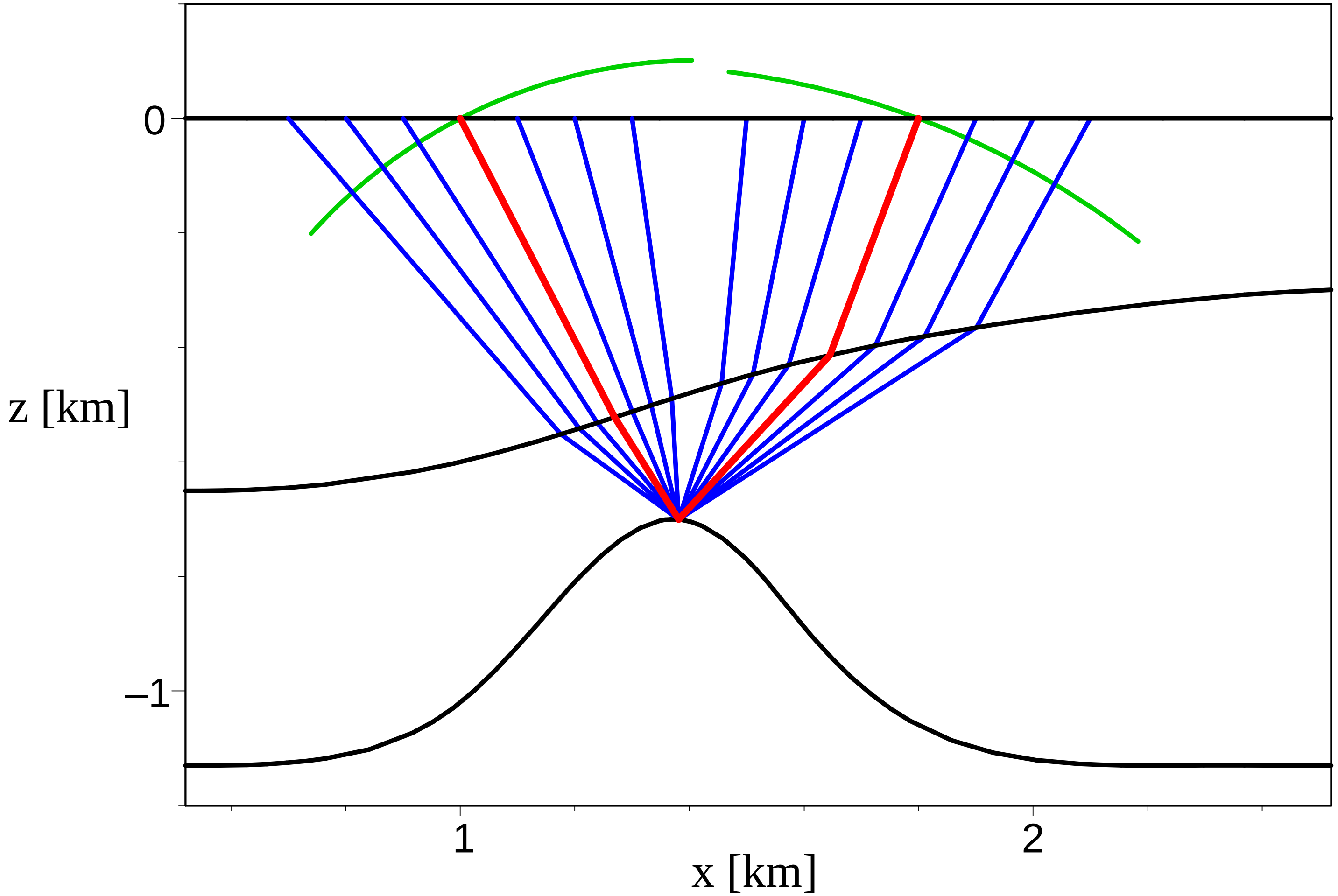
☞ presentation by Duveneck tomorrow ☜

Finite-offset case

Wavefronts for generalized Stereotomography



☞ presentation by Bergler and Hubral today ☜

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Applications of attributes

Based on approximation of diffraction traveltimes:

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Based on approximation of diffraction traveltimes:

- approximation of geometrical spreading factor

Applications of attributes

Based on approximation of diffraction traveltimes:

- approximation of geometrical spreading factor
- approximation of projected Fresnel zone

Applications of attributes

Based on approximation of diffraction traveltimes:

- approximation of geometrical spreading factor
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- data-driven time migration

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Extensions based on attribute extrapolation at surface:

Applications of attributes

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Extensions based on attribute extrapolation at surface:

- CRS stack with topography

Applications of attributes

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  - direct use of source and receiver elevations

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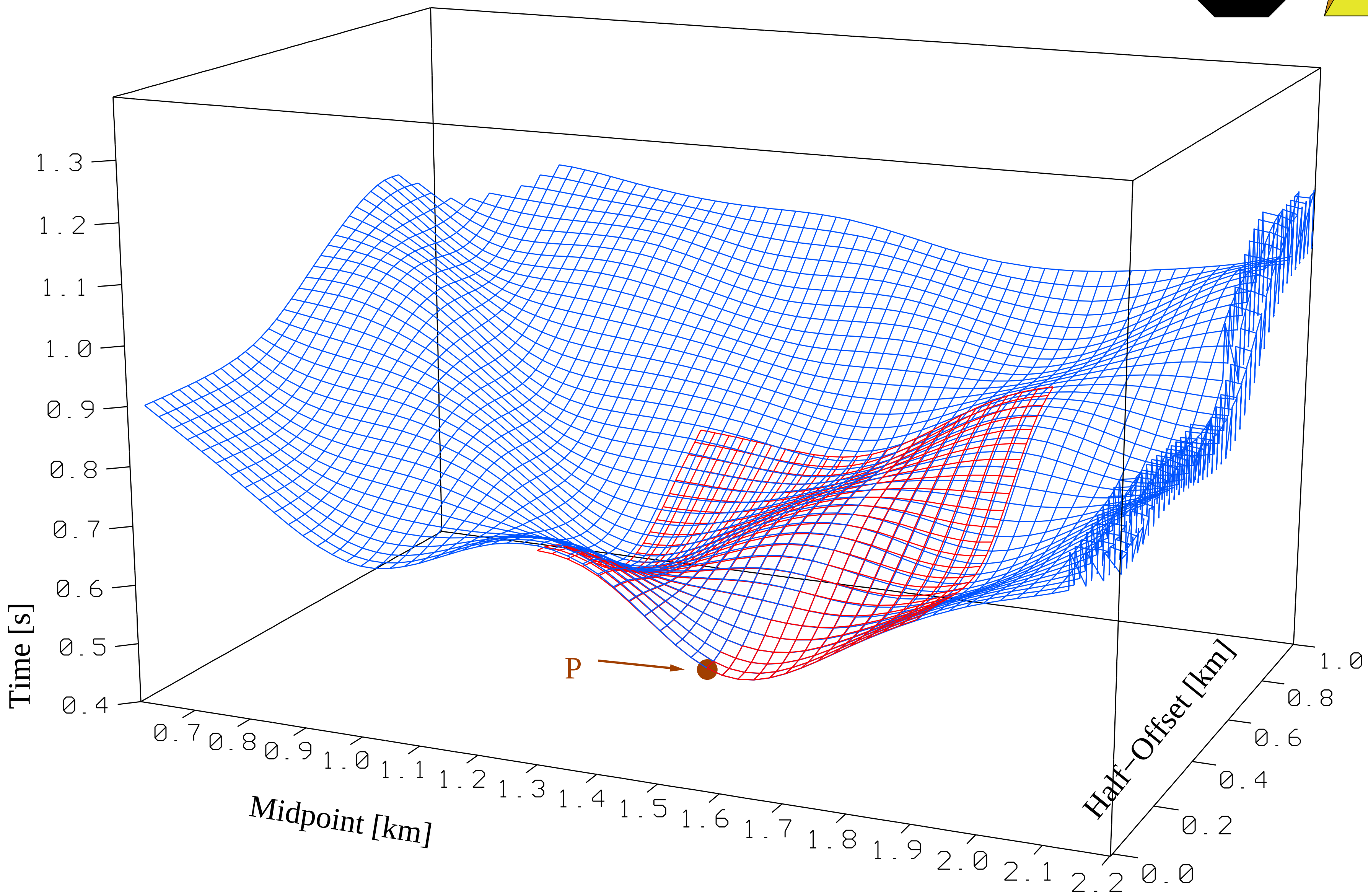
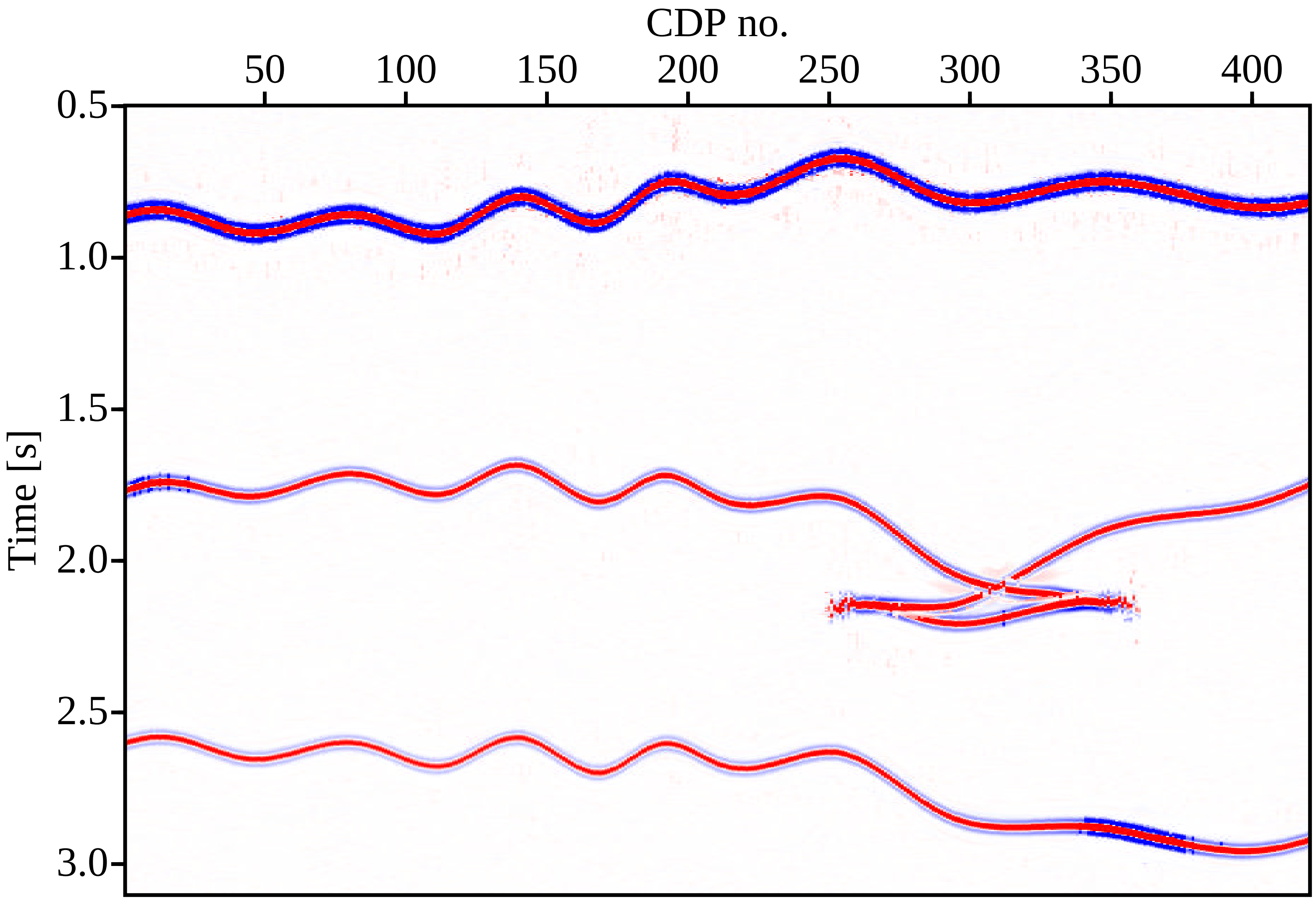
Applications of attributes



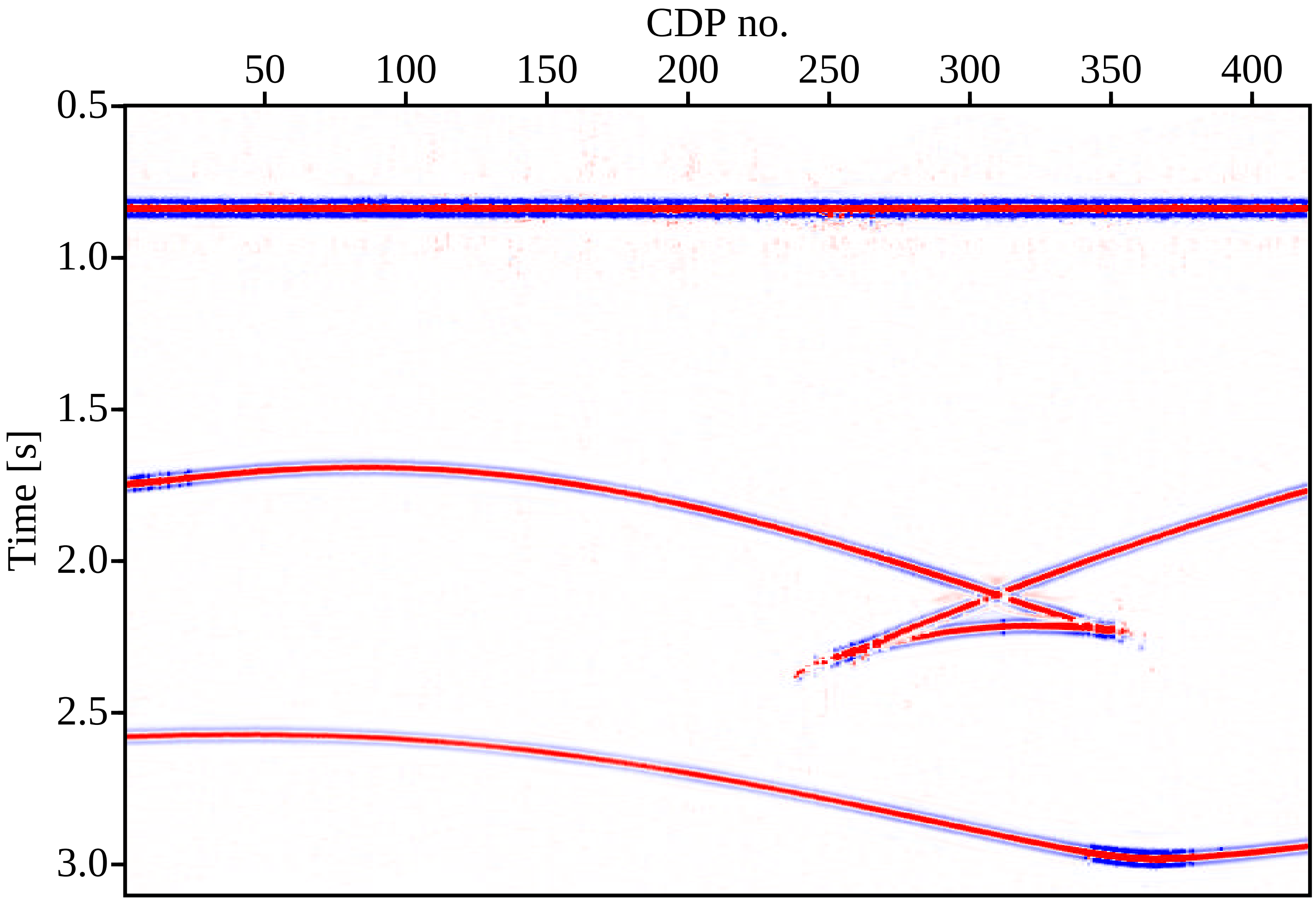
Synthetic example with topography

Applications of attributes



Optimized CRS stack

Applications of attributes



Redatumed CRS stack section

Conclusions

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Acknowledgments

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